Functional Programming
Lecture 1

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Introduction

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- It does not depend on a programming language but some languages are more suitable for functional programming than others.
- Functional programming languages are languages encouraging usage of pure functions.

Pure functions

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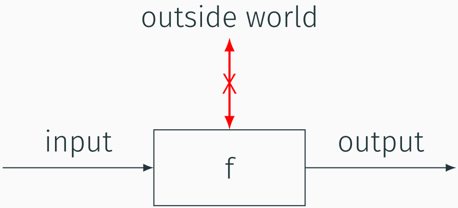

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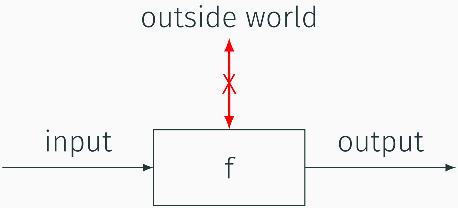

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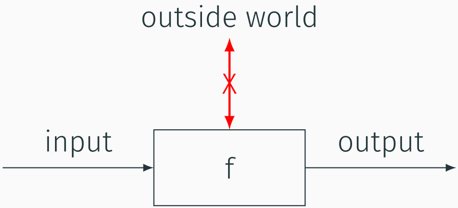

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A pure functional program = a composition of pure functions

Examples of (im)pure functions

```python
counter = 0

def pure(x, y):
  return (x + y)/2

def do_other(x):
  global counter
  counter += 1
  return x**2

def depends_on_other(x):
  return counter + x**2
```

- unit testing and debugging

Advantages

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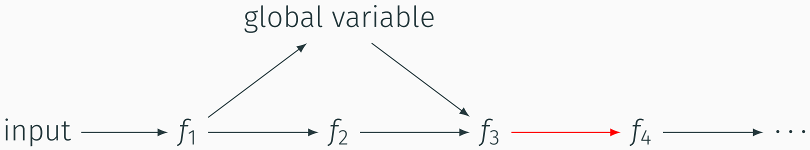

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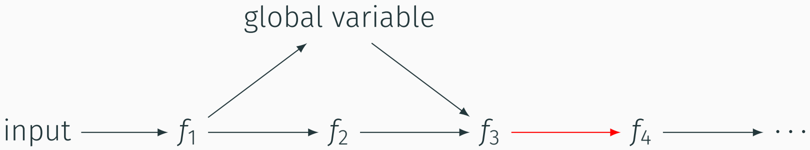

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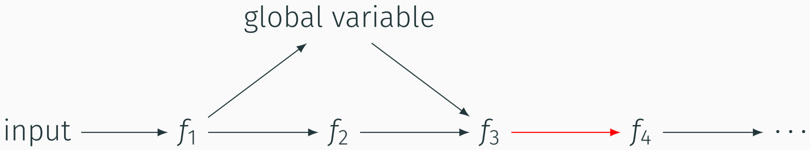

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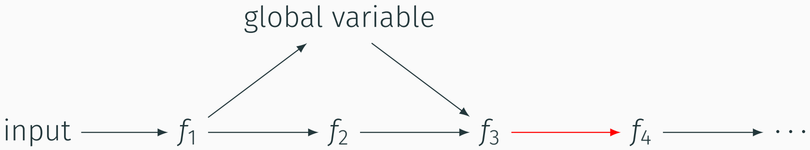

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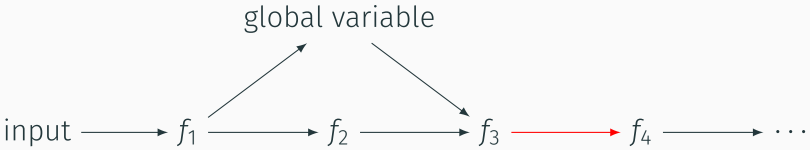
- concurrency and parallelism — $f(g_1, \ldots, g_n)$
- formal verification — mathematical induction, algebraic reasoning
- compiler optimization, pure functions are cachable



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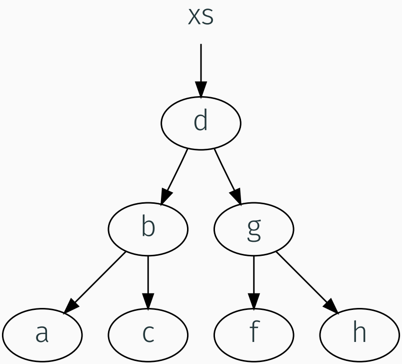
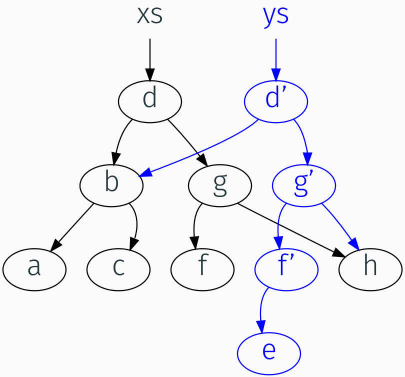
- To reduce the number of copying, persistent data structures are used.

Persistent data structures



```
ys = insert ("e", xs)
```



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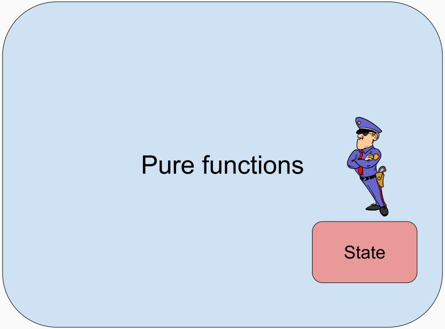

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Theorem

Turing machines and λ-calculus are equally strong regarding computing functions.

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  - strictly separates the pure core from the mutable shell

Suggested literature

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- DrRacket: `racket-lang.org`
  text editor + REPL (read-evaluate-print loop)

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  ```
  ; This is a one-line comment
  #|
      This is
      a block comment
  |#
  ```

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S-expression

```
(fn arg1 arg2 ... argN)
```

Definitions

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- Nested definitions

```
(define (name a1 ... aN)
   (define (fn b1 ... bM) <body-fn>)
   <body-using-fn>)
```

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More precisely, we subsequently evaluate subexpressions until we end up with the expression's value.

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- Evaluation of some syntactic forms is lazy if, cond, and, or

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(cond [(odd? 12) 1]
      [(even? 12) 2]
      [else 3])     => 2
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- Other types:
  `symbol?, pair?, procedure?, vector?, port?`

Simple debugging

- Helper print-outs

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(begin (displayln x)
       <do-work>)
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- Tracing function calls and returns

```
(require racket/trace)
(trace fn)
(untrace fn)
```

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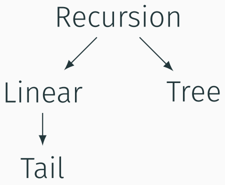

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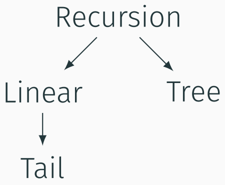

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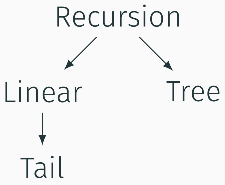

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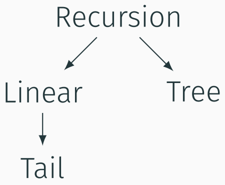

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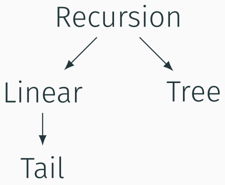

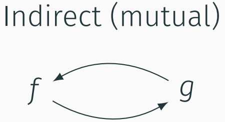

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Examples

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(define (loop) (loop))
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(define (fact n)
    (if (<= n 1)
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(define (loop) (loop))

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Not space efficient. It needs $O(n)$ memory.

Example — Tail recursion

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This needs $O(1)$ memory due to tail elimination.

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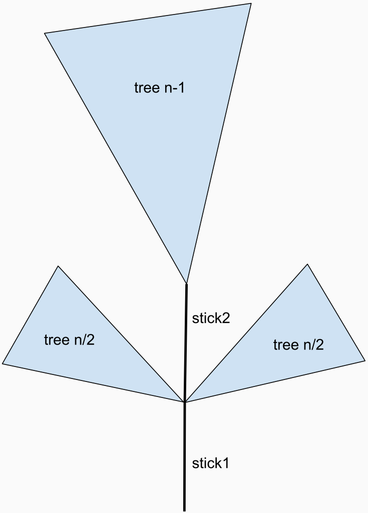
5. Draw the fractal of size $n - 1$ in the direction $d + 5$.

Fractal example



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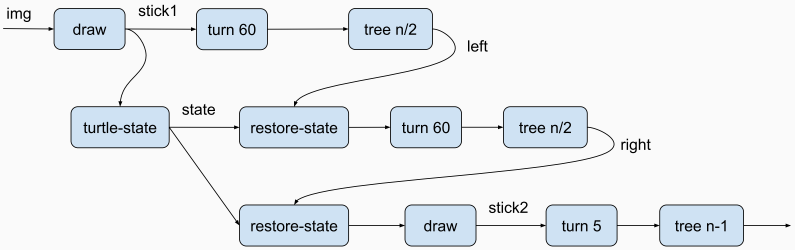

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- Functional languages handle iterative computations by recursion.
- We classify recursive functions according to the number of recursive calls they make on linear-recursive and tree-recursive functions.
- Tail recursive functions are space efficient as they do not consume memory by making recursive calls.